Graphing $y = ax^2 + c$

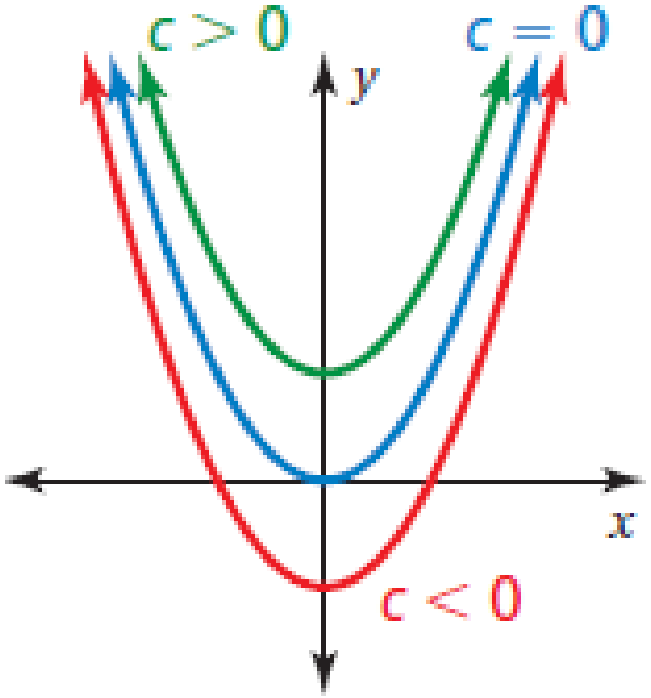
Lesson 8.3

Key Idea

Graphing $y = x^2 + c$

- When $c > 0$, the graph of $y = x^2 + c$ is a vertical translation c units up of the graph of $y = x^2$.

- When $c < 0$, the graph of $y = x^2 + c$ is a vertical translation $|c|$ units down of the graph of $y = x^2$.



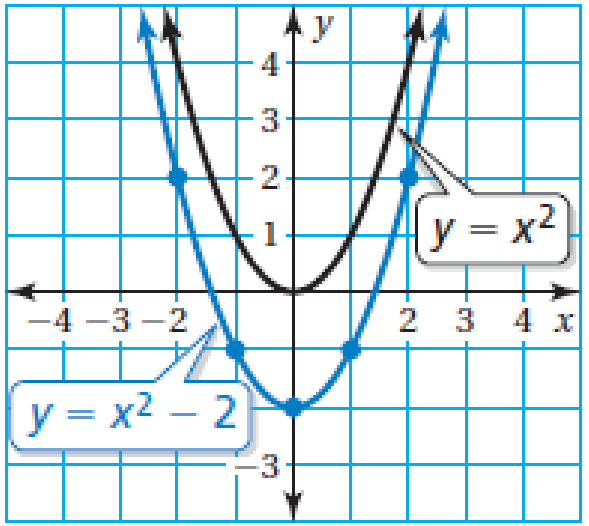
EXAMPLE 1 Graphing $y = x^2 + c$

Graph $y = x^2 - 2$. Compare the graph to the graph of $y = x^2$.

Step 1: Make a table of values.

x	-2	-1	0	1	2
y	2	-1	-2	-1	2

Step 2: Plot the ordered pairs.

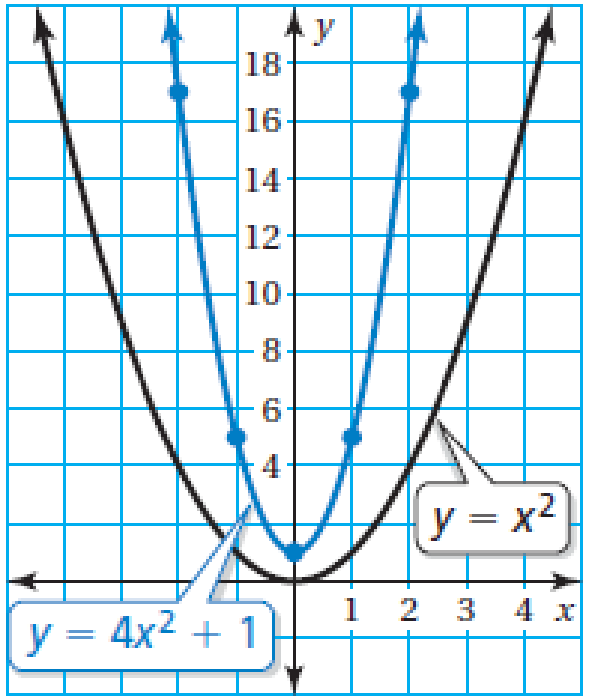


∴ Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = x^2 - 2$ is a translation 2 units down of the graph of $y = x^2$.

EXAMPLE 2 Graphing $y = ax^2 + c$

Graph $y = 4x^2 + 1$. Compare the graph to the graph of $y = x^2$.

x	-2	-1	0	1	2
y	17	5	1	5	17



∴ Both graphs open up and have the same axis of symmetry, $x = 0$. The graph of $y = 4x^2 + 1$ is narrower than the graph of $y = x^2$. The vertex of the graph of $y = 4x^2 + 1$ is a translation 1 unit up of the vertex of the graph of $y = x^2$.

EXAMPLE 3 Translating the Graph of $y = x^2 + c$

Which of the following is true when you translate the graph of $y = x^2 - 4$ to the graph of $y = x^2 + 5$?

(A) The graph shifts 1 unit up.

(B) The graph shifts 9 units up.

(C) The graph shifts 9 units down.

(D) The graph shifts 1 unit down.

Both graphs open up and have the same axis of symmetry, $x = 0$. The vertex of $y = x^2 - 4$ is (0, -4). The vertex of $y = x^2 + 5$ is (0, 5). To move the vertex from (0, -4) to (0, 5), you must translate the graph 9 units up.

The correct answer is B.

A **zero** of a function $f(x)$ is an x-value for which $f(x) = 0$. A zero is located at the x-intercept of the graph of the function.

In other words, a **"zero" of a function** is an input value (x) that produces an output(y) of **zero**.

An alternative name for such a point (x, 0) in this context is an x-intercept.

Examples:

a. $y = x^2 - 9$

$0 = x^2 - 9$

$0 = (x + 3)(x - 3)$

$x + 3 = 0 \quad or \quad x - 3 = 0$

$x = -3 \quad or \quad x = 3$

Zeros are -3, 3

b. $y = -x^2 + 16$

$0 = -x^2 + 16$

$0 = x^2 - 16$

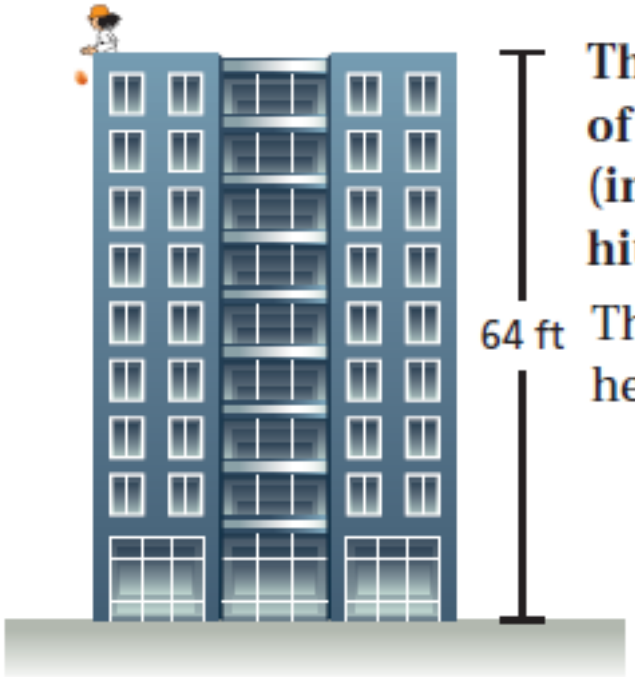
$0 = (x + 4)(x - 4)$

$x + 4 = 0 \quad or \quad x - 4 = 0$

$x = -4 \quad or \quad x = 4$

Zeros are -4, 4

EXAMPLE 4 Real-Life Application



The function $f(t) = -16t^2 + s_0$ gives the approximate height (in feet) of a falling object t seconds after it is dropped from an initial height s_0 (in feet). An egg is dropped from a height of 64 feet. When does the egg hit the ground?

The initial height is 64 feet. So, the function $f(t) = -16t^2 + 64$ gives the height of the egg after t seconds. It hits the ground when $f(t) = 0$.

Step 1: Make a table of values and sketch the graph.

t	0	1	2
$f(t)$	64	48	0

Step 2: Find the zero of the function.

$$0 = -16(t^2 - 4)$$

$$0 = -16(t + 2)(t - 2)$$

$$t + 2 = 0 \quad or \quad t - 2 = 0$$

$$t = -2 \quad or \quad t = 2$$

The zero is 2.

The egg hits the ground 2 seconds after it is dropped.